Lecture 1

Some Graphs on the xy −plane

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Equations of Straight Lines

An equation in the form

$$Ax + By + C = 0$$

where A, B and C are any constant and A and B are not both zero is called a first degree of equation or a linear equation in x and y, its graph is a straight line.

Conversely, any straight line can be represented by a first−degree equation in x and y.

$$Ax + By + C = 0 \quad \text{or} \quad By = -Ax - C$$

$$y = -\frac{A}{B}x - \frac{C}{B}, \qquad \text{let } \; m = -\frac{A}{B}, b = -\frac{C}{B}, \qquad \text{thus } \; y = mx + b.$$

Suppose $b = -mx_1 + y_1$, where x_1 and y_1 are any constant then

$$y = mx - mx_1 + y_1, \qquad \text{thus} \qquad y - y_1 = m(x - x_1).$$

$$Ax + By + C = 0 \quad \text{or} \quad \frac{Ax}{-C} + \frac{By}{-C} = 1 \rightarrow \frac{x}{-\frac{C}{A}} + \frac{y}{-\frac{C}{B}} = 1$$

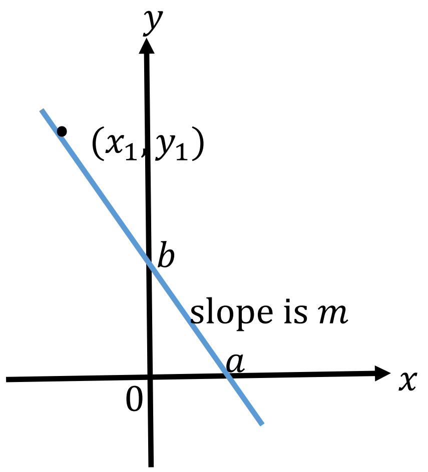
Suppose $a = -\frac{C}{A}, b = -\frac{C}{B}$ here we get $\frac{x}{a} + \frac{y}{b} = 1.$

Equation of lines may be written in several different forms:

Slope $-$intercept form $\quad y = mx + b$

Point$-$slope form $\quad y - y_1 = m(x - x_1)$

Double $-$ intercept form $\quad \dfrac{x}{a} + \dfrac{y}{b} = 1$

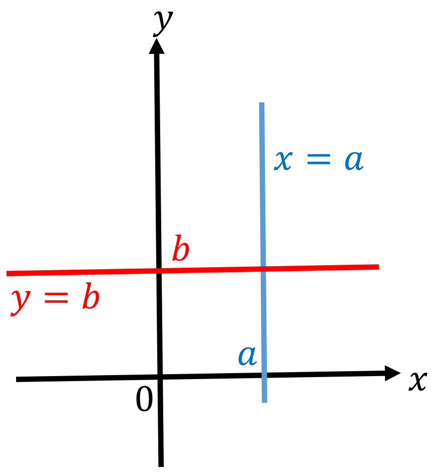


where m is the slope of the line, a is the x $-$intercept,

b is the y $-$intercept and (x_1, y_1) is any point on the line.

The vertical line with x $-$intercept a is $\quad x = a$

The horizontal line with y $-$intercept b is $\quad y = b$



Angle of inclination

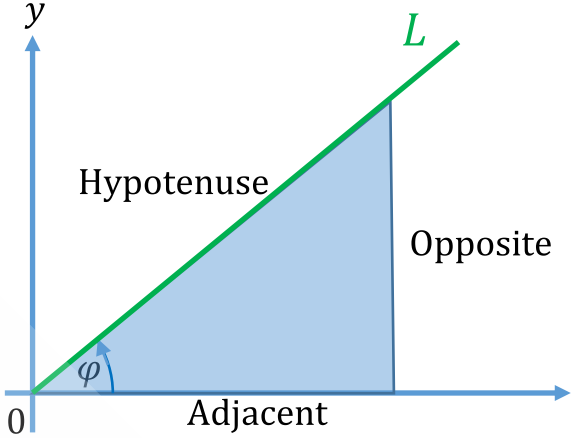
The slope of a non−vertical line L is related to the angle φ that L makes with the positive x −axis. If φ is the smallest positive angle measured counterclockwise from the x −axis to L then the slope of L is

$$m = \frac{\text{Opposite}}{\text{Adjacent}} = \tan \varphi$$

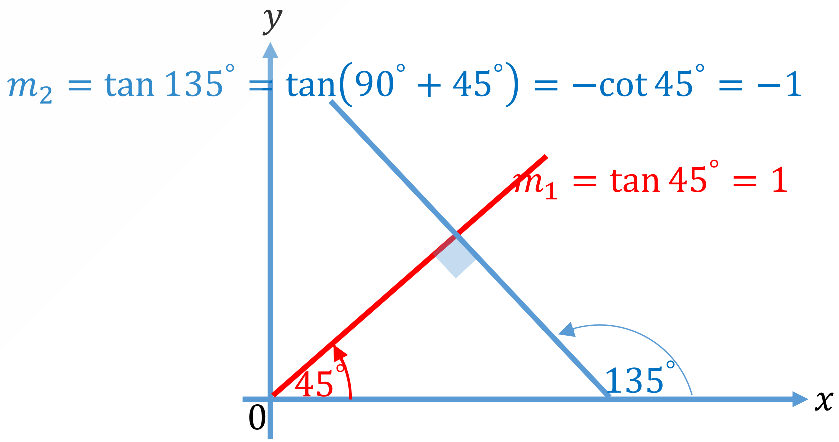
m is slope, φ is angle of inclination.

If $$y = mx + b$$

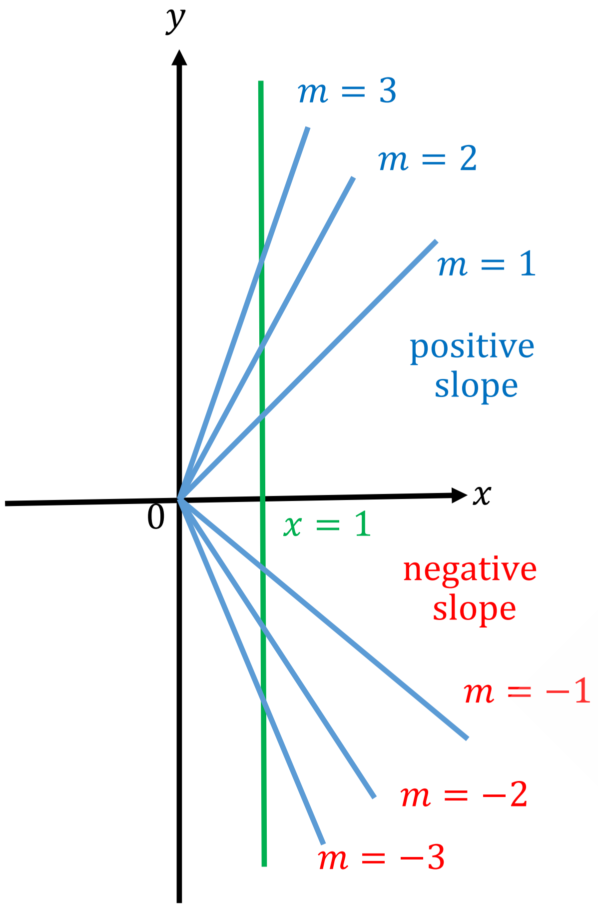
then $$\frac{dy}{dx} = m = \tan \varphi.$$

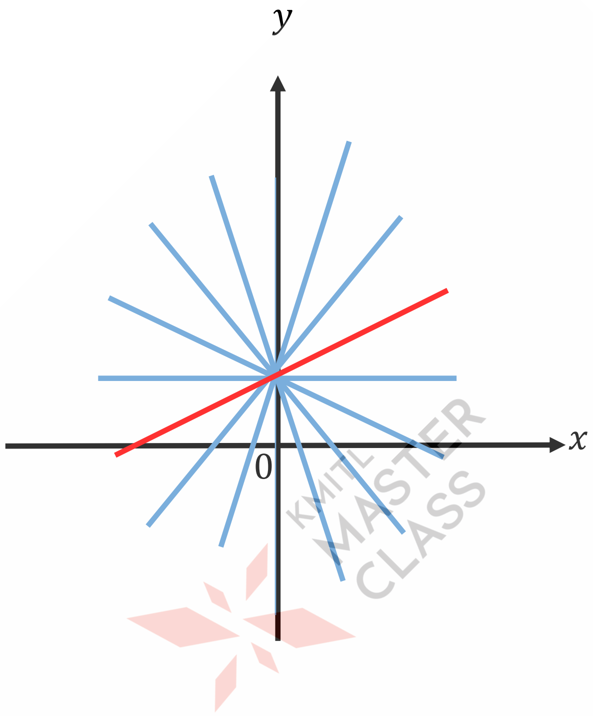


$$m = \frac{\text{Opposite}}{\text{Adjacent}} = \tan \varphi.$$

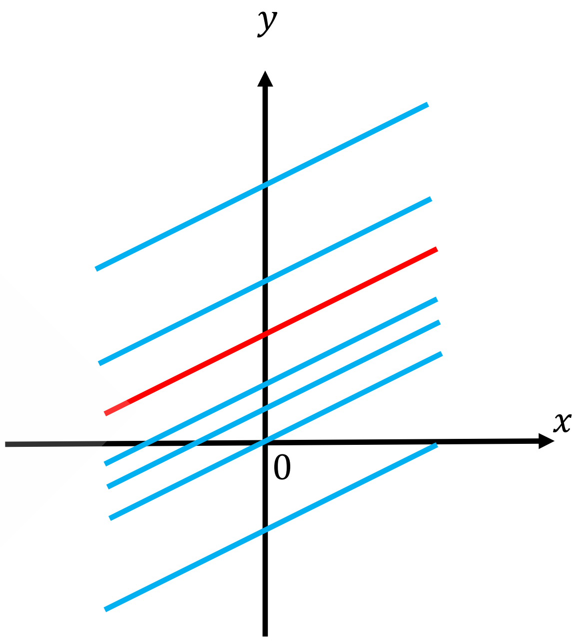


If $L_1 \perp L_2$ then $m_1 m_2 = -1$



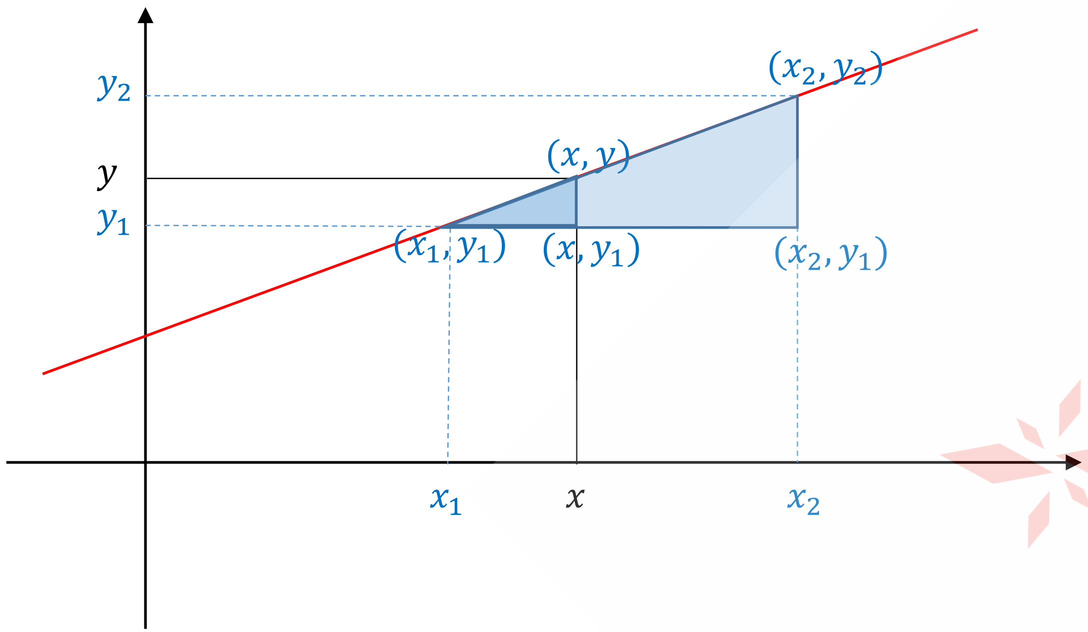


$y = mx + b$
b fixed and m vary



$y = mx + b$
m fixed and b vary

Given two points (x_1, y_1) and (x_2, y_2). Find an equation of a straight line passes through the two given points.



$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - y_1 = m(x - x_1), m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y = mx + (y_1 - mx_1)$$

$$y = mx + b, b = y_1 - mx_1$$

Example 1 Find equation of the straight line passes through the point $(2,3)$ with slope $-\frac{3}{2}$ and find the x −intercept and the y −intercept.

Solution By point−slope form $\quad y - y_1 = m(x - x_1)$.

Since $x_1 = 2, y_1 = 3$ and $m = -\frac{3}{2}$ we get

$$y - 3 = -\frac{3}{2}(x - 2).$$

So that $y = -\dfrac{3}{2}x + 6 \quad$ or $\quad 3x + 2y = 12 \quad$ or $\quad \dfrac{x}{4} + \dfrac{y}{6} = 1$

Let $x = 0$ then $y = 6$, let $y = 0$ then $x = 4$.

Thus, the x −intercept is 4 and the y −intercept is 6. ■

The point that the straight intercepts the x −axis is $(4,0)$, intercepts the y −axis is $(0,6)$

Example 2 Find the equation of the straight line that passes through the points $(-2,-1)$ and $(3,4)$, then find the slope and the angle of inclination.

Solution The slope of any line is $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

The slope of this line is $$m = \frac{4-(-1)}{3-(-2)} = \frac{-1-4}{-2-3} = 1$$

By point–slope form $$y - y_1 = m(x - x_1)$$

Let $(x_1, y_1) = (-2,-1)$ we get $y - (-1) = 1(x - (-2))$ or $y = x + 1.$

Let $(x_1, y_1) = (3,4)$ we get $y - (4) = 1(x - 3)$ or $y = x + 1.$

Either of the two given points gives the same equation of the straight line.

The slope of any line is $m = \tan\varphi$ then the angle of inclination is $\varphi = \tan^{-1} m$.

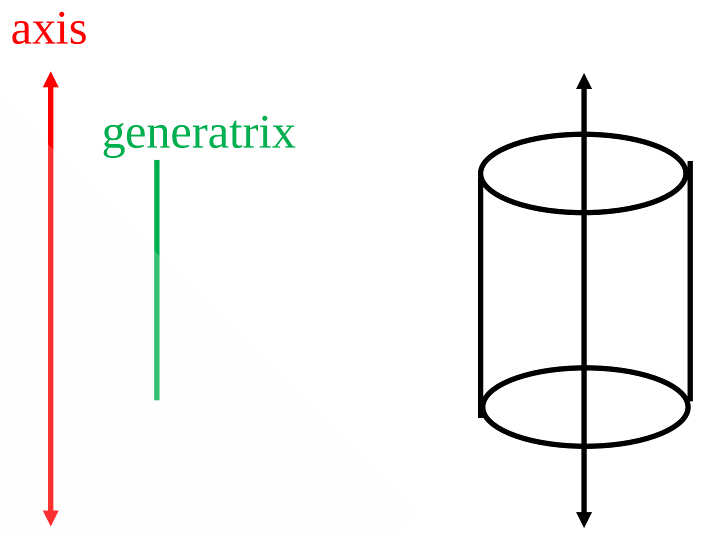
The angle of inclination of this line is $\varphi = \tan^{-1} 1 = \frac{\pi}{4}$. ■

Surfaces of revolution

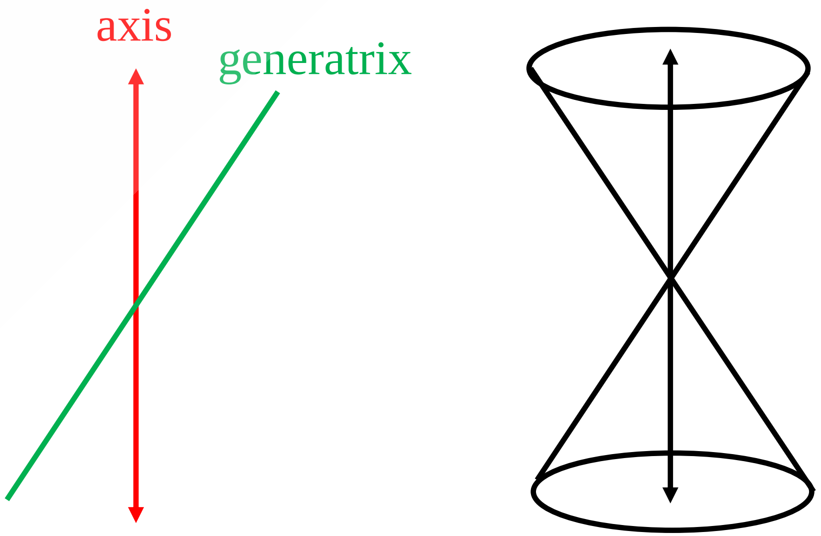
A surface of revolution is a surface generated by rotating a curve called generatrix around an axis of rotation.

The surface generated by a straight line parallel to the axis of rotation is called cylinder. The axis of rotation is the axis of cylinder.

If the straight line is intersected to the rotating axis, the surface is called double cone.



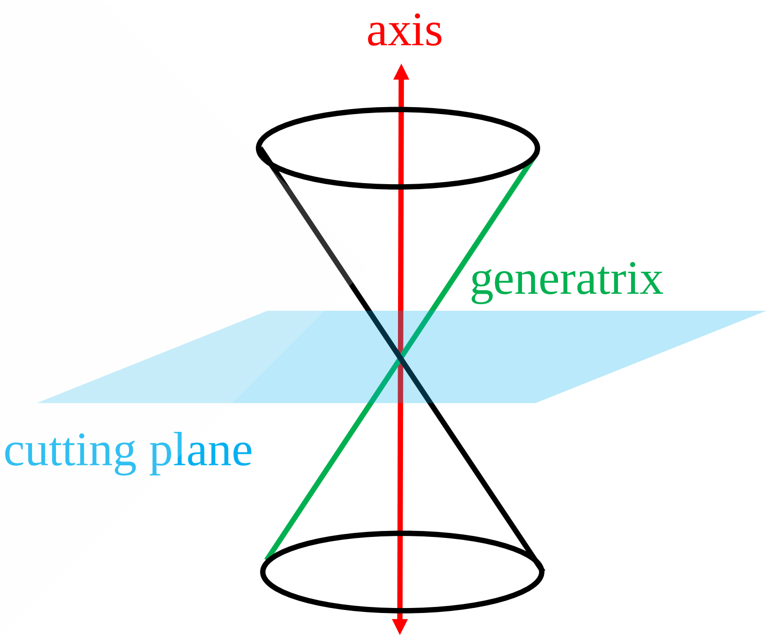
Cylinder



Double cone

Definition of conic sections

A conic section is a curve obtained by the intersection the surface of a double cone with a plane, called the cutting plane. The four types of conic section are circle, ellipse, hyperbola and parabola.

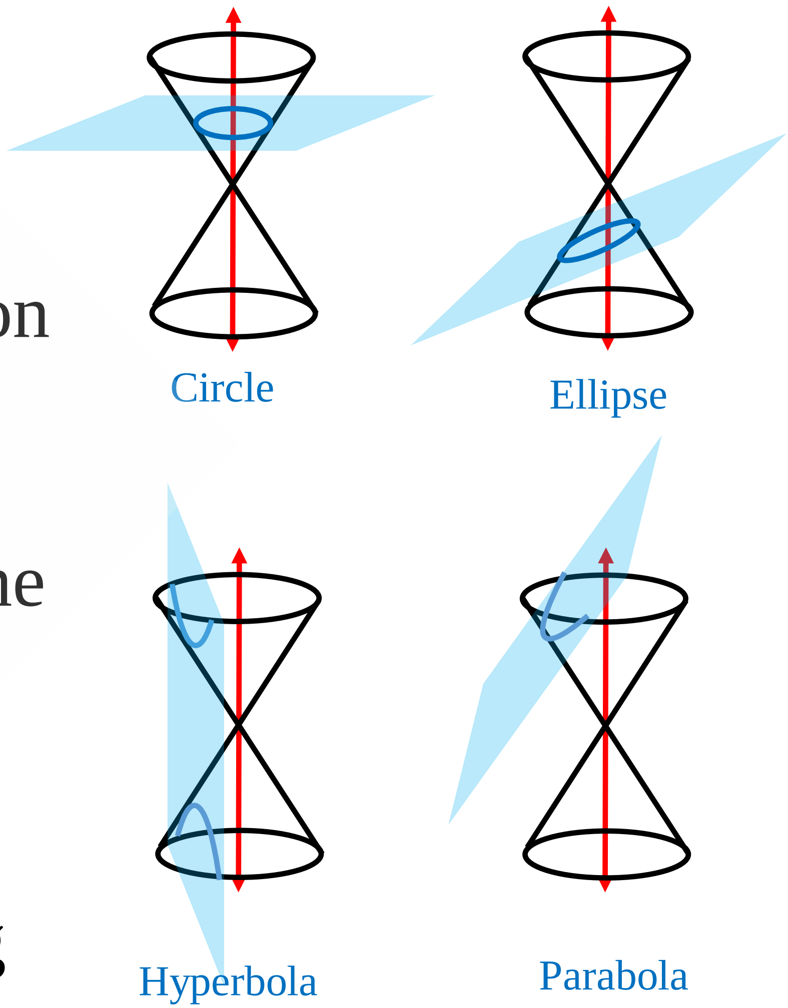


The circle is obtained when the cutting plane is perpendicular to the axis of the cone.

The ellipse is obtained when the cutting plane making some angle with the axis of the cone and the intersection is a closed curve.

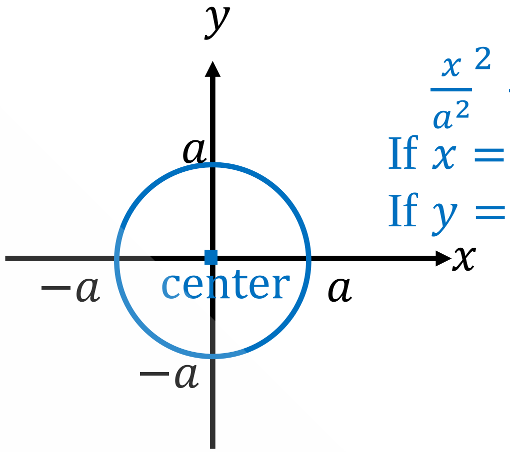
If the cutting plane is intersected both halves of the cone and parallel to the axis of the cone, producing the two separated unbounded curves called hyperbola.

If the cutting plane is parallel to exactly one generating curve of the cone, then the conic is unbounded and is called parabola.



A **circle** is the locus of all points equidistant from a central point.

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$$



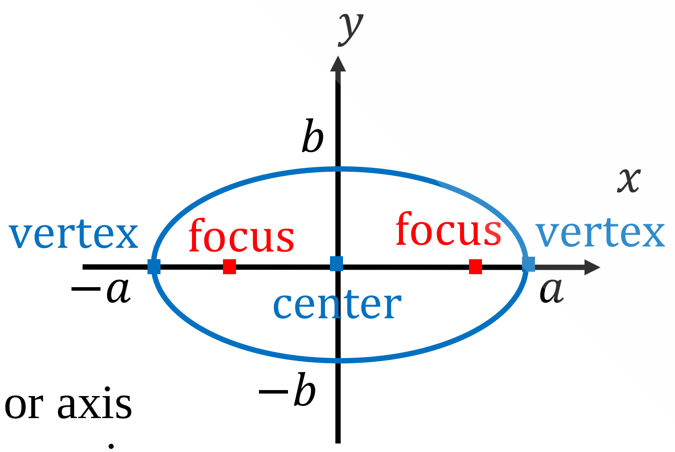
$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$

If $x = 0$ then $y = \pm a$

If $y = 0$ then x$= \pm a$

An **ellipse** is a plane curve surrounding two **focal points**, such that for all points on the curve, the sum of the two distances to the focal points is a positive constant. The **vertex** of the ellipse are the point where the ellipse intersect the **focal axis** or the major axis.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



$a > b$

oblate ellipse

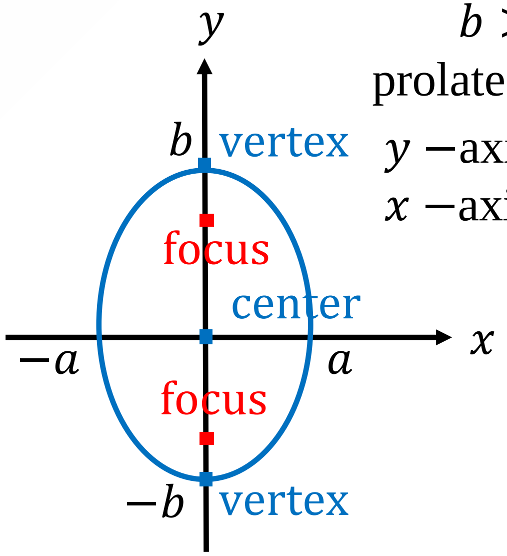
x −axis is major axis

y −axis is minor axis

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

If $x = 0$ then $y = \pm b$

If $y = 0$ then x$= \pm a$



$b > a$

prolate ellipse

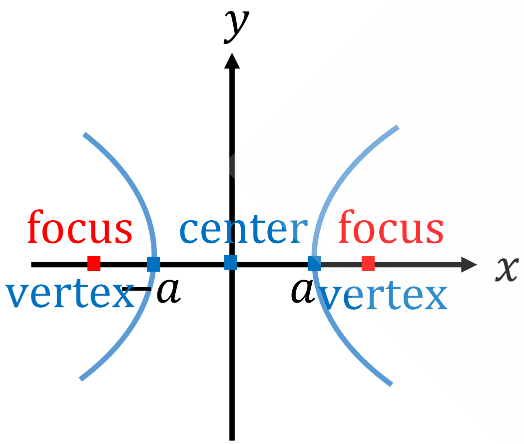
y −axis is major axis

x −axis is minor axis

A **hyperbola** is a plane curve surrounding two **focal points**, such that for all points on the curve, the difference of the two distances to the focal points is a positive constant. The **vertex** of the hyperbola are the points where the hyperbola intersect the **focal axis**.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

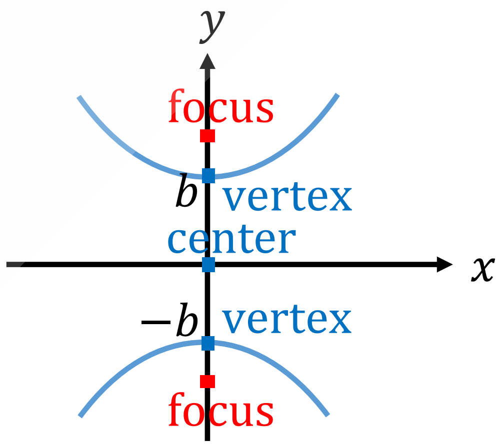
$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

If $y = 0$ then $x = \pm a$

If $x = 0$ then y is not any real number.



$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

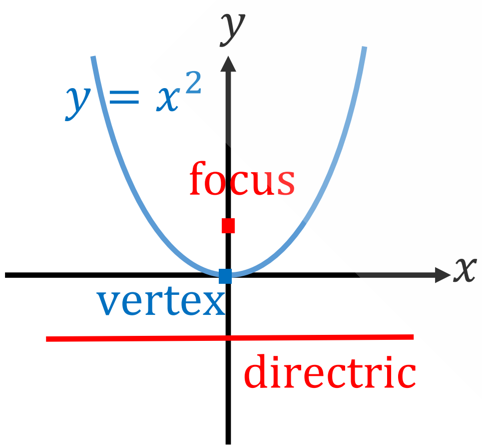
If $x = 0$ then $y = \pm b$

If $y = 0$ then x is not any real number.

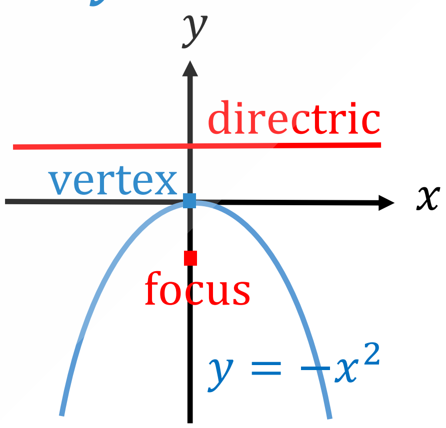
The **parabola** is defined as the **locus** of a point which moves so that it is always the same distance from a fixed point (called the **focus**) and a given line (called the **directrix**).

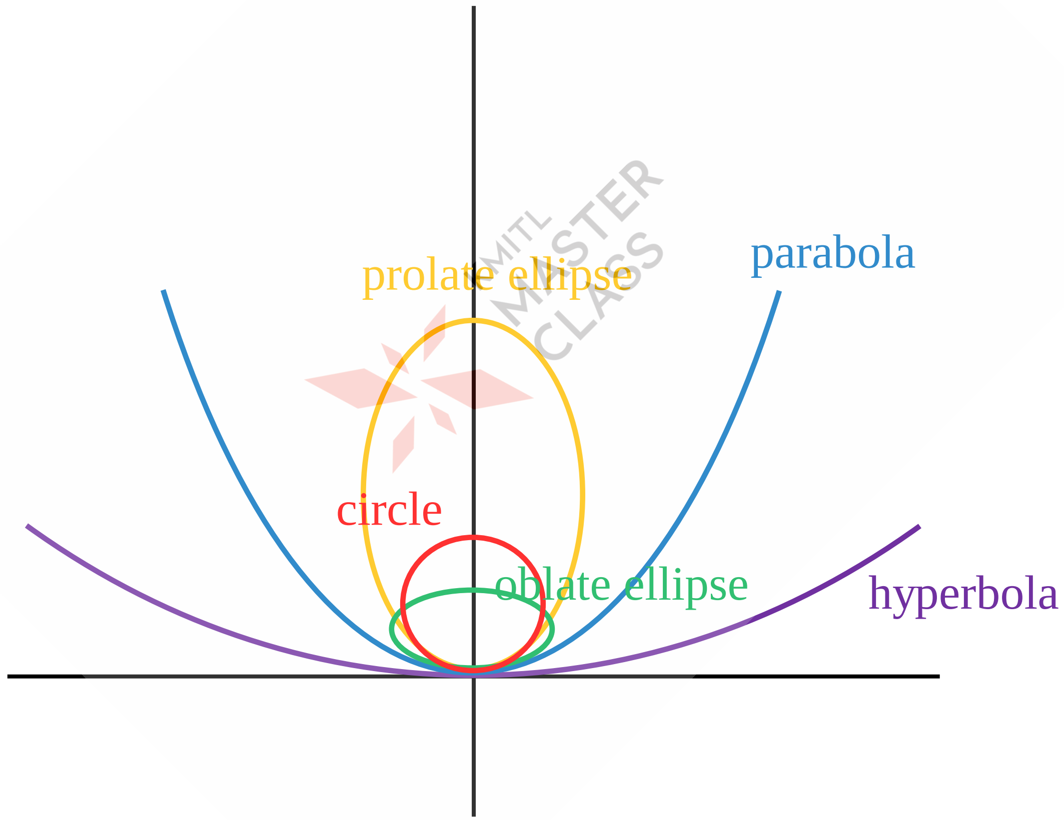
The **vertex** of the parabola is the point where the parabola intersect the axis of symmetry, here it is y −axis.

$y = x^2$



$y = -x^2$




prolate ellipse
parabola
circle
oblate ellipse
hyperbola

Example 3

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

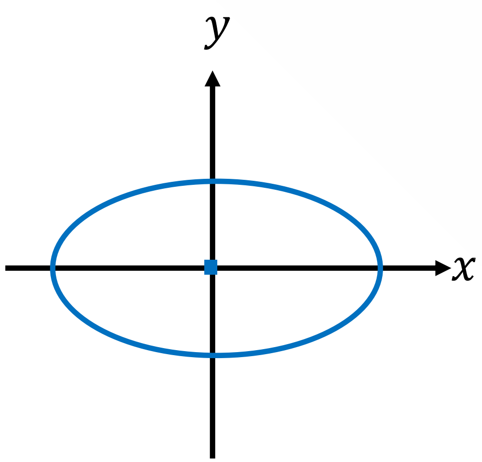
Name : Oblate Ellipse

Center : (0,0)

Major axis : x −axis

Minor axis : y −axis

Vertex : (-4,0) and (4,0)



$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

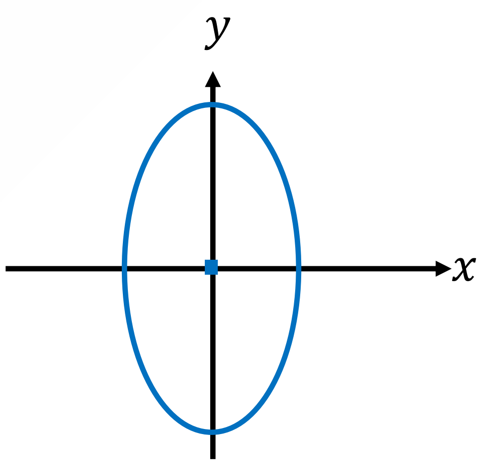
Name : Prolate Ellipse

Center : (0,0)

Major axis : y −axis

Minor axis : x −axis

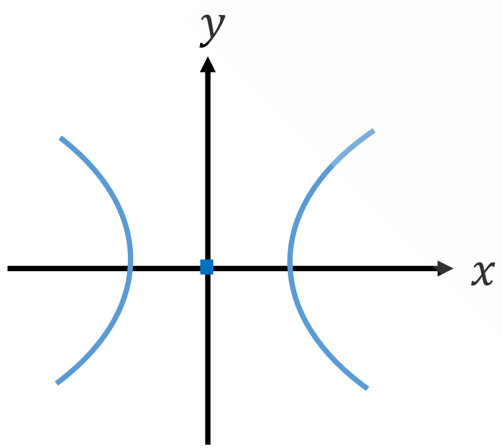
Vertex : (0,-4) and (0,4)



Example 4

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Name : Hyperbola
Center : (0,0)
Major axis : x −axis
Vertex : (−4,0) and (4,0)



$$\frac{y^2}{9} - \frac{x^2}{16} = 1$$

Name : Hyperbola
Center : (0,0)
Major axis : y −axis
Vertex : (0,−3) and (0,3)